Analytical Synthesis of Aircraft Control Laws♣

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Abstract
This paper contains some results related to the creation of solutions of complex system theory problems in an analytical form. It makes it possible theoretically to take into account the whole aggregate of determinatives and to determine most of the small effects that together influence system behaviour. This approach is not intended to replace current approaches using numeric calculations; however, it does offer a powerful supportive and supplemental technique. The intelligent combination of analytical and numerical approaches opens the door to new ideas for system theory and for system design of control systems that are complex yet safe.

1. Introduction

The main advances of system theory made in the last century are concerned with significant developments of numerical methods for modelling, analysis and design. Many problems in many important fields such as development of high technologies, effective engineering systems, high-precision navigation of the sub miniature mechanisms and spacecraft, have been solved by burdening computers with huge amounts of computation.

It may be expected that at least in the near future the progress of human activity will be entirely related to improved computational methods of system theory. However, current computational methods used in analysis and design have inherent disadvantages of which the most important are as follows:

- ✓ Impossibility of generalization of the results obtained (no matter how large the number calculated situations, in formal terms one deals with nothing more than a set of individual realizations);
- ✓ Impossibility of constructing sets of equivalent solutions (no numerical methods exist to generate alternative solutions, though the problem under consideration may have such);
- ✓ Impossibility of proving the negative outcome of the problem (its basic insolvability does not follow from the lack of solution of each of the realizations checked);
- ✓ Impossibility of strict determination of the problem bottlenecks that exert a cardinal influence on its major properties or characteristics.

These disadvantages are overcome to a varied extent by increasing the number of numerical experiments (coverage) and resorting to experts' opinions (heuristics). Understandably, the potentialities of each of these techniques are rapidly exhausted with increased complexity of the problem under consideration.

A newly formed field of research has been developed and christened "System Embedding Technology" and its component parts are illustrated in Figure 1. An approach to modelling, analysis and design mainly of linear dynamic systems is presented. It focuses on an advanced analytical solution of the mathematical problems that arise. Study of the solvability conditions and construction of formulaic representations of the entire set of possible equivalent solutions relies on specially developed constructs which include the formation and use of so-called *pro-matrices* (problem matrices) and a method of canonization of arbitrary matrices which separates the linearly dependent and linearly independent rows and columns of these matrices.

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Individual fragments of the technology have already been seen in publications of other authors, but the authors of this paper are not aware of any previous work that directly precedes the results generalized in the recently published book [1].

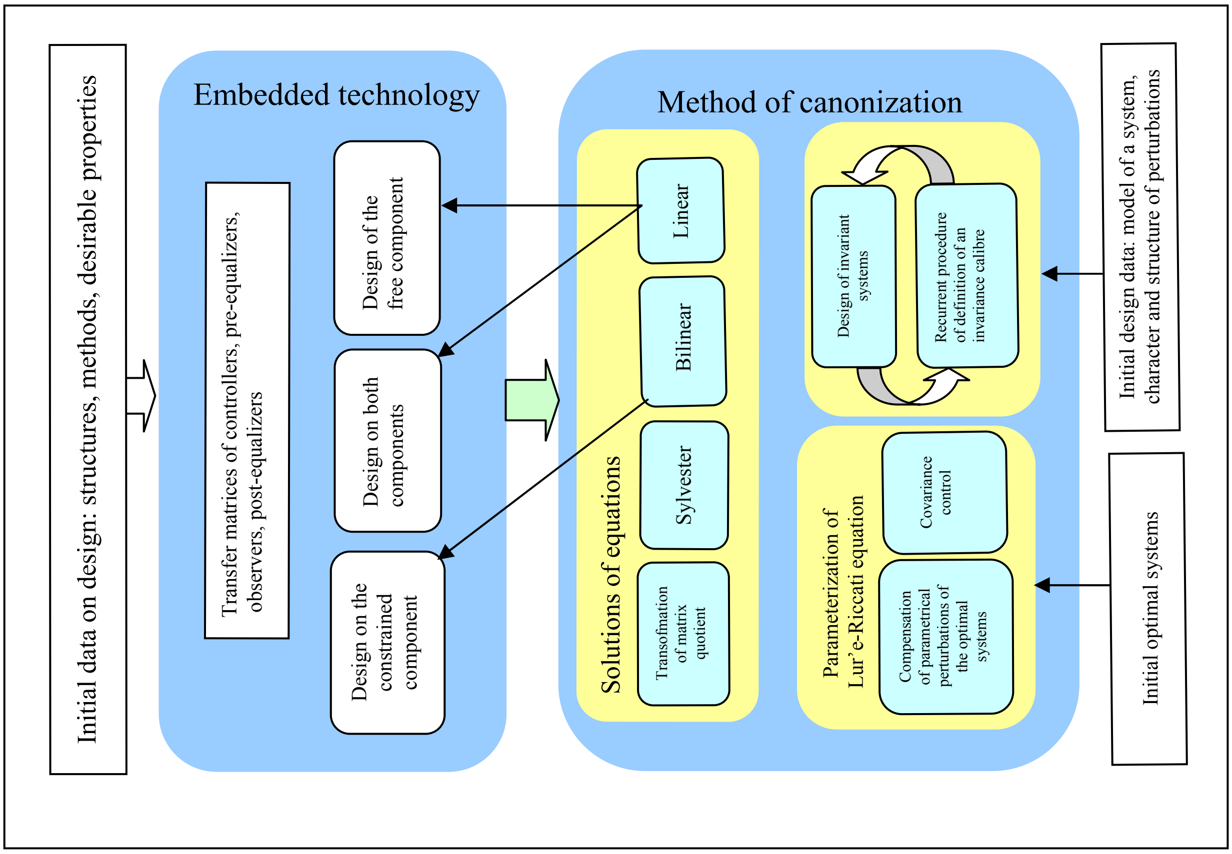

Figure 1: General structure of the field of research

In practice the meaning of using a rigorously developed set of equivalent solutions is about:

- ✓ Selection of a preferred variant from the set of equivalent solutions which takes account various non-formal requirements (initially ignored at the early phase of problem of synthesis of control of system motion);
- ✓ Implementation of several equivalent solutions providing an opportunity to support reconfiguration (required by partial faults and errors) of on-board control systems; especially reconfigurations of a special type: i.e. where structural changes of control **do not** impact the dynamic features of the system, for example in an air-craft-control system.

2. Method of Matrix Canonization

To provide the basis for analytic research and new solutions of matrix algebraic equations an original method was developed [2], and called the "method of matrix canonization", it is based on the representation of a matrix of any size with rank r

$$A \to (\overline{A}^L, \widetilde{A}^L, \widetilde{A}^R, \overline{A}^R) \qquad (1)$$

Where a non unique set of left $\overline{A}^L$ and right $\overline{A}^R$ divisors of zero of maximum rank, as well as left $\widetilde{A}^L$ and right $\widetilde{A}^R$ canonizations, together satisfy identity

$$\begin{bmatrix} \widetilde{A}^L \\ \overline{A}^L \end{bmatrix} A \begin{bmatrix} \widetilde{A}^R & \overline{A}^R \end{bmatrix} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}. \qquad (2)$$

At the same time we use a summary canonization and left $(B)^L$ and right $(B)^R$ unity devisors of matrix B:

$$\widetilde{A} = (A)^{\sim} = \widetilde{A}^R \widetilde{A}^L, \qquad (B_{m\times n})^L B_{m\times n} = I_n, \qquad B_{m\times n}(B_{m\times n})^R = I_m. \tag{3}$$

One of possible options for practical canonization of a matrix makes use of elementary transformations of rows and columns of an initial matrix using the following scheme. A map-case is formed by adding at the left and at the bottom single matrixes. By elementary transformation this map-case is converted to the form when the matrix at right top corner becomes combination of one single and three zero blocks:

$$\begin{array}{c|c} I_m & A_{m\times n} \\ \hline & I_n \end{array} \quad \to \quad \begin{array}{c|c} \begin{bmatrix} \widetilde{A}^L_{r\times m} \\ \overline{A}^L_{(m-r)\times m} \end{bmatrix} & \begin{bmatrix} I_r & 0_{r\times(n-r)} \\ 0_{(m-r)\times r} & 0_{(m-r)\times(n-r)} \end{bmatrix} \\ \hline & \begin{bmatrix} \widetilde{A}^R_{n\times r} & \overline{A}^R_{n\times(n-r)} \end{bmatrix} \end{array}, \tag{4}$$

in that case initially single matrices I_m and I_n will be replaced by matrices that have different divisors of zero of maximum range $\overline{A}^L$, $\overline{A}^R$ and canonozators $\widetilde{A}^L$, $\widetilde{A}^R$.

The possibility for analytic use of this scheme is defined primarily by three conditions. The first condition is connected with the fact that formulas of repeatable canonization, while objects, are not initial matrices but results of their canonization at the previous iteration. These formulas are presented in the Table 1 below.

Table 1: Formula summary of matrix canonization

Initial matrix	Result of canonization		
	Left zero devisor	Summary canonizator	Right zero devisor
A	$\overline{A}^L$	$\widetilde{A}^R\widetilde{A}^L$	$\overline{A}^R$
$\overline{A}^L$	0	$(\overline{A}^L)^R$	$A\widetilde{A}^R$
$\overline{A}^R$	$\widetilde{A}^L A$	$(\overline{A}^R)^L$	0
$\widetilde{A}^L$	0	$A\widetilde{A}^R$	$\overline{\widetilde{A}^L}^R$
$\widetilde{A}^R$	$\overline{\widetilde{A}^R}^L$	$\widetilde{A}^L A$	0
$\widetilde{A}$	$\overline{\widetilde{A}^R}^L$	A	$\overline{\widetilde{A}^L}^R$

The second condition is about obtaining equations for matrix multiplication:

$$AB \to \left(\overline{AB}^L, (AB)^{\sim}, \overline{AB}^R\right) =$$

$$= \left(\begin{bmatrix} \overline{\overline{B}^L \overline{A}^R}^L \overline{B}^L \widetilde{A} \\ \overline{A}^L \end{bmatrix},\ \widetilde{B}(I_n - \overline{A}^R(\overline{B}^L\overline{A}^R)^{\sim}\overline{B}^L)\widetilde{A},\ \begin{bmatrix} \widetilde{B}\overline{A}^R \overline{\overline{B}^L\overline{A}^R}^R & \overline{B}^R \end{bmatrix} \right) \tag{5}$$

and the third condition is when the matrix is blocked, for example with vertical blocks

$$A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \to \begin{pmatrix} \overline{A}^L \\ \widetilde{A} \\ \overline{A}^R \end{pmatrix} = \begin{pmatrix} \begin{bmatrix} \overline{A}^L & 0 \\ -\overline{A_2\overline{A}_1^R}^L A_2\widetilde{A}_1 & \overline{A_2\overline{A}_1^R}^L \end{bmatrix} \\ \overline{A}_1^R \overline{A_2\overline{A}_1^R}^R \\ \begin{bmatrix} (I_n - \overline{A}_1^R(A_2\overline{A}_1^R)^{\sim}A_2)\widetilde{A}_1 & \overline{A}_1^R(A_2\overline{A}_1^R)^{\sim} \end{bmatrix} \end{pmatrix}. \tag{6}$$

By taking into account symmetry similar formulas can be obtained for the case of division of the matrix into blocks that placed horizontally. Even the fact that these formulas may seem at first sight to be rather cumbersome in practice it is not hard to apply them to real problems.

3. Analytical Solution of Equations of System Theory

By using such a canonization of matrices the conditions of solvability and equations for complete solutions of many equations of system theory have been obtained [2 – 4]. In addition to traditional linear equations special attention has been given to bilinear equations that were practically abandoned by specialists because of their previous intractability. Table 2 below outlines the solution methods applicable in the circumstances shown.

Table 2: An analytical solution of linear equations of a system theory

Equation	Notation	Solvability conditions	Solution formulas
Left-side	$AX = B$	$\overline{A}^L B = 0$	$\{\mathring{X}\}_{\mu} = \tilde{A}B + \overline{A}^R \mu$
Right-side	$XA = B$	$B\overline{A}^R = 0$	$\{\mathring{X}\}_{\eta} = B\tilde{A} + \eta\overline{A}^L$
Double-sided	$AXC = B$	$\overline{A}^L B = 0,\ B\overline{C}^R = 0$	$\{\mathring{X}\}_{\mu,\eta} = \tilde{A}B\tilde{C} + \overline{A}^R\mu + \eta\overline{C}^L$
Enveloping	$AX + YB = C$	$\overline{A}^L C\overline{B}^R = 0$	$\{\mathring{X}\}_{S,\mu,\eta} = \tilde{A}S + \overline{A}^R\mu + \eta B$, $\{\mathring{Y}\}_{S,\rho,\eta} = (C-S)\tilde{B} + \rho\overline{B}^L - A\eta$, $\{S\}_{\pi} = A[\tilde{A}C\overline{B}^R(\overline{B}^R)^L + \pi B]$
Sylvester	$AX - XB = C$	$\{\lambda_i\}_A \cap \{\lambda_j\}_B$	$\{\mathring{X}\}_{\kappa_1,\dots,\kappa_k} = \sum_{i=1}^{k} \overline{A-\lambda_i I_m}^R \kappa_i \overline{B-\lambda_i I_n}^L$
Similarity	$A' = TAT^{-1}$	$\{\lambda_i\}_A = \{\lambda_j\}_{A'}$	$\{T\}_{\kappa_1,\dots,\kappa_k} = \sum_{i=1}^{k} \overline{A'-\lambda_i I_n}^R \kappa_i \overline{A-\lambda_i I_n}^L$
Symmetrical	$AX + X^{\mathrm{T}}A^{\mathrm{T}} = Q$	$\overline{A}^L Q(\tilde{A}^L)^{\mathrm{T}} = 0,$ $\overline{A}^L Q(\overline{A}^L)^{\mathrm{T}} = 0$	$\{\mathring{X}\}_{\eta,\mu} = \tilde{A}[\frac{1}{2}Q + A(\eta - \eta^{\mathrm{T}})A^{\mathrm{T}}] + \overline{A}^R\mu$

Table 3: An analytical solution of bilinear equations of a system theory

Equation	Notation	Solvability conditions	Solution formulas
Pure	$XY = B$	$n \geq \mathrm{rank}\, B$	$\{\mathring{X}\}_{T,\pi} = \begin{bmatrix} B\tilde{B}^R & \pi \end{bmatrix} T$, $\{\mathring{Y}\}_{T,\mu} = T^{-1}\begin{bmatrix} \tilde{B}^L B \\ \mu \end{bmatrix}$, $\pi\mu = 0$
With factors	$XAY = B$	$\mathrm{rank}\, A \geq \mathrm{rank}\, B$	$\{\mathring{X}\}_{T,\pi,\rho} = \begin{bmatrix} B\tilde{B}^R & \pi \end{bmatrix} T\tilde{A}^L + \rho\overline{A}^L$, $\{\mathring{Y}\}_{T,\mu,\varphi} = \tilde{A}^R T^{-1}\begin{bmatrix} \tilde{B}^L B \\ \mu \end{bmatrix} + \overline{A}^R\varphi$
Mixed	$XAY + XC = B$	$n \geq \mathrm{rank}B, \quad \exists T,\pi,\mu:$ $\overline{A}^L\left(T^{-1}\begin{bmatrix} \tilde{B}^L B \\ \varphi \end{bmatrix} - C\right) = 0$	$\{\mathring{X}\}_{T,\eta} = \begin{bmatrix} B\tilde{B}^R & \eta \end{bmatrix} T$, $\{\mathring{Y}\}_{T,\varphi,\mu} = \tilde{A}\left(T^{-1}\begin{bmatrix} \tilde{B}^L B \\ \varphi \end{bmatrix} - C\right) + \overline{A}^R\mu$
Set	$XAY + XC = B,$ $XAZ = D$	$\mathrm{rank}\, D \leq \mathrm{rank}\, A, \quad n \geq \mathrm{rank}B,$ $\exists T,N,\varphi,\eta,\psi: \quad \overline{A}^L\left(T^{-1}\begin{bmatrix} \tilde{B}^L B \\ \varphi \end{bmatrix} - C\right) = 0,$ $\begin{bmatrix} B\tilde{B}^R & \eta \end{bmatrix} T = \begin{bmatrix} D\tilde{D}^R & \psi \end{bmatrix} N$	$\{\mathring{X}\}_{T,\varphi,\eta} = \begin{bmatrix} B\tilde{B}^R & \eta \end{bmatrix} T$, $\{\mathring{Y}\}_{T,\varphi,\vartheta} = \tilde{A}\left(T^{-1}\begin{bmatrix} \tilde{B}^L B \\ \varphi \end{bmatrix} - C\right) + \overline{A}^R\vartheta$, $\{\mathring{Z}\}_{N,\kappa,\xi} = \tilde{A}N^{-1}\begin{bmatrix} \tilde{D}^L D \\ \kappa \end{bmatrix} + \overline{A}^R\xi$
Lur'e	$A^{\mathrm{T}}\mathrm{X} + \mathrm{X}A -$ $- \mathrm{X}G\mathrm{X} = -Q$	$\mathrm{rank}G \geq \mathrm{rank}\underbrace{(Q + A^{\mathrm{T}}\tilde{G}A)}_{\mathrm{H}}$, $\left(\begin{bmatrix} H\tilde{H}^R & \pi \end{bmatrix} T\tilde{G}^L\right)^{\mathrm{T}} = \tilde{G}^R T^{-1}\begin{bmatrix} \tilde{H}^L H \\ \mu \end{bmatrix}$	$\{\mathring{\mathrm{X}}\}_{T,\mu,\varphi} = \tilde{G}A + \overline{G}^R\varphi -$ $- \tilde{G}^R T^{-1}\begin{bmatrix} (Q + A^{\mathrm{T}}\tilde{G}A)^{\sim L}(Q + A^{\mathrm{T}}\tilde{G}A) \\ \mu \end{bmatrix}$

The need to solve these kinds of systems of equations relates with the task of deliberately ignoring the behaviour of the system when given non-zero initial conditions. In practice this is done in many theoretical and applied problems.

When considering the solution of matrix equations two important components must always be considered:

- ✓ The test of solvability for the equation;
- ✓ The formulaic representation, in parametric form, of all possible solutions (for all cases where solvability is satisfied).

Some of linear and bilinear equations with solvability conditions and solutions are presented in the Tables 2 and 3. In the table equations in curly brackets define equivalent elements. At the same time lower index of these brackets define the elements in the generated variations for all elements of corresponding set.

6. Localization (search) of Faults

General aviation aircraft are not particularly well supported by on-ground safety maintenance. The Principle of Active System Safety (PASS) aims to monitor and predict the condition of the aircraft including structure, engines, avionics and pilot to avoid accidents, or reduce the level of possible harm during operational use in real time of flight [5, 6]. This will involve continuous real time analysis of flight data as well as data previously accumulated from flights of a particular aircraft.

Using the result of analysis of real system one might create a digraph of faults appearance and propagation. This analysis assumes that each node represents subsystem, unit or element of the system as it is shown on Figure 2.

Here some nodes are elements with possible faults, others might be considered as internal processes with manifestation of faults. Thus where is input and where is output one might define only by number of node. For example, nodes 1, 2, 4, 6, 7 and 8 are elements of the system, they might have faults; nodes 3, 5, 10 and 11 are elements of system that might manifest discrepancy (i.e. fault manifestation) mentioned faults and, respectively, 9 is an internal element that does not belong neither to first nor the second group and presents logic of processes.

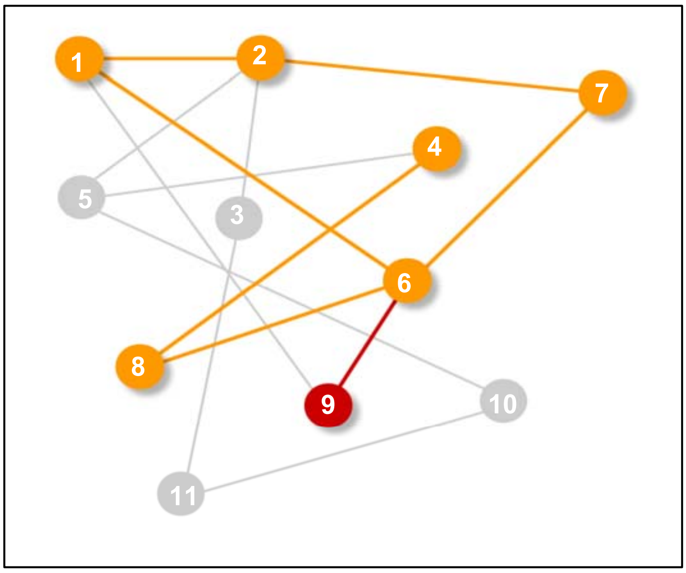

Figure 2: Graph of dependencies between elements

Then one has to introduce a matrix E called the Output Matrix. This matrix defines nodes of a mixed graph that corresponds to observable faults. Denote: a vector X is an aggregate of all nodes, a vector Y is an aggregate of fault manifestation where the value 0 represents an absence of connection and the value 1 represents the existence of a connection or influence. In general faults might manifest via some linear combination of graph state. As an example considers the matrix of outputs is defined as equalities as below:

$$\begin{bmatrix} y_3 \\ y_5 \\ y_{10} \\ y_{11} \end{bmatrix} = E \begin{bmatrix} x_1 \\ \vdots \\ x_{11} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_{E} \begin{bmatrix} x_1 \\ \vdots \\ x_{11} \end{bmatrix}. \tag{7}$$

Some algorithm aimed to localize the system faults must include at the least tree procedures:

- generation an initial estimated value for vector of state $\hat{X}(0)$, which contains:
  0 if the corresponding element is guaranteed working;
  1 if the corresponding element is exhibiting a fault;
  * if the state is undefined, state of the element is impossible to determine using observable manifestation of faults;
- improving of estimated value with any interactive calculation: $\hat{X}(1)$, $\hat{X}(2)$ and so on;
- decision-making about potential faulty elements based on analysis of an improved estimate of $\hat{X}(k)$.

Now let us consider just the first step, the estimation of a value $\hat{X}(0)$. This procedure might be described as looking for the whole set of solutions for an equation:

$$Y(0) = EX(0), \quad E \in \Re^{m \times n}, \quad m < n, \tag{8}$$

on the vector $X(0)$. In the general case this solution might be presented as:

$$\left\{\hat{X}(0)\right\}_{\mu} = \tilde{E}Y(0) + \overline{E}^{R}\mu,$$

where curly brackets denote the set of indistinguishable solutions, generated by variation of a parameter μ, in this case μ is vector of dimension $n - \mathrm{rank}E$; $\tilde{E}$ is canonizator of matrix E; $\overline{E}^{R}$ is the right divider of zero for matrix E with maximum rank, i.e. a matrix of dimension $n \times (n - \mathrm{rank}E)$ with maximum rank, for which the following condition holds:

$$E\overline{E}^{R} = 0.$$

Canonizator of a matrix $\tilde{E}$ in the problems here (all elements of matrix E are represented only by «0» and «1», matrix itself has maximal row rank) are equal to the transposed value of the initial matrix: $\tilde{E} = E^{\mathrm{T}}$. Thus resulting formulae has a form

$$\left\{\hat{X}(0)\right\}_{\mu} = E^{\mathrm{T}}Y(0) + \overline{E}^{R}\mu, \tag{9}$$

and all elements of the vector μ are then presented as «*».

7. A Simple Example

To check the suggested solution let us consider a simplified scheme of height and speed parameters of aircraft shown on Figure 3.

As a source of faults one might consider:

- two TPP together with related pipes of full pressure (TPP1, TPP2);
- two pairs of sensors of static pressure together with two looped pipes of static (SH1, SH2);
- one Air Data System (ADS).

Output (fault manifestation) here is aggregate of devices indicated parameters:

– airspeed indicator AI (y_1);

– altimeter A (y_2);

– variometer V (y_3).

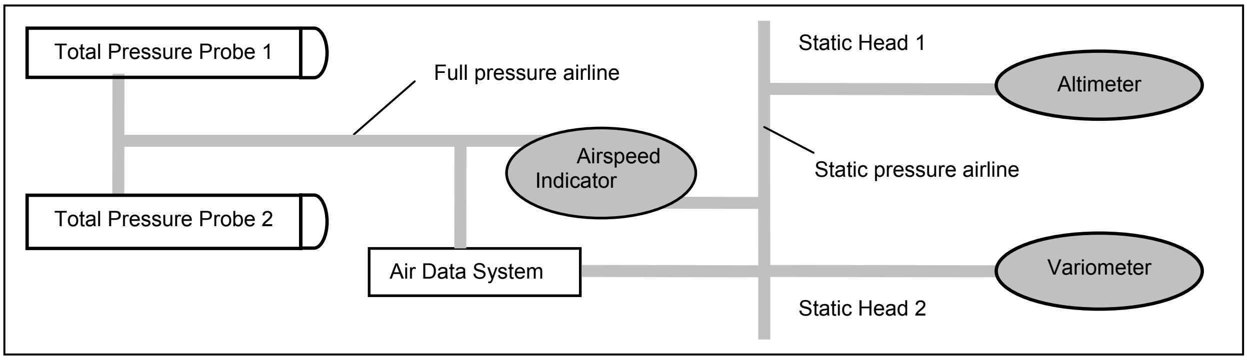

Figure 3: Scheme for Aircraft Aerometry

Let us use notations: x_1 is output of TPP1; x_2 is output of TPP2; x_3 is state of the tract (pipe) of full pressure; x_4 is output of AI; x_5 is output of ADS; x_6 is output of A; x_7 is output of V; x_8 is state of the pipe of static pressure; x_9 is output of SH1; x_{10} is output of SH2. So the scheme of Figure 3 corresponds to a formula

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}}_{E} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{bmatrix}. \tag{10}$$

In this simple case, for matrix with elements “0” and “1”, the canonization of the matrix E gives us following matrices

$$\tilde{E} = E^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \overline{E}^{R} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{11}$$

Let the fault be, for example, icing then the pressure sensors TPP1 and TPP2 may be blocked by ice. Due to this fault required pressure does not develop in the full pressure pipe, devices AI and ADS show wrong values. This leads us to the fact that after several iterations in the model vector X in the model of «matrix» gets the value

$$X^{\mathrm{T}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{12}$$

At the same time fault manifest as device AI is not working:

$$Y^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}. \tag{13}$$

Using solution (9) gives value for vector

$$\hat{X}^{\mathrm{T}}(0) = \begin{bmatrix} * & * & * & 1 & * & 0 & 0 & * & * & * \end{bmatrix} \quad (14)$$

The above mentioned approach is followed by an iterative procedure of suspect elimination by means replacing "*" by "0" or "1" if it is possible, but this is too difficult to go into here and will be the topics of a separate report.

Here we only note that during further steps of an iterative algorithm an estimation $\hat{X}$ of a vector X has a final result

$$\hat{X}(k) = \hat{X}(2) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & * & * \end{bmatrix}. \quad (15)$$

Comparing this value with real value (12) one might discover that all faulty elements have been found. Due to the duplication of TPP1 and TPP2 it is not necessary to produce a conclusion about state of both sensors at the same time. Therefore positions 9 and 10 have *. In fact the * shows that checking system does not have enough information to completely define the states of these two elements.

Conclusion

An approach to modelling, analysis and design mainly of the linear dynamic systems is presented. It focuses on an advanced analytical solution which is applicable to the mathematical problem that has arisen. Study of the solvability conditions and construction of the formulaic representations of the entire set of possible equivalent solutions relies on a specially developed construct. This includes a method of canonization of the arbitrary matrices which separates the linearly dependent and linearly independent rows and columns of these matrices.

Aviation is the most complex area for the application of technological advances: it has the most complex and long lasting working periods, an extremely wide range working environments and needs multi-disciplinary skills from personnel involved. Therefore implementation of new approaches is becoming a subject of multifaceted research covering many logically connected aspects of mathematics and engineering in the pursuit of operationally reliable system designs.

This research may be considered as a new part of project ON-Board Active Safety System (ONBASS) funded by EU Grant No 516045. The problem is described by the following: to implement a search procedure for a faulty element (localization, using well-known terminology) assuming multiple faults of aircraft equipment. Analyzing a simple example based on aircraft equipment demonstrates the efficiency of proposed approach.

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